

# Extraction of singlet states from non-interacting high-dimensional spins

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We present a scheme for the extraction of singlet states of two remote particles of arbitrary quantum spin number. The goal is achieved through post-selection of the state of interaction mediators sent in succession. A small number of iterations is sufficient to make the scheme effective. We propose two suitable experimental setups where the protocol can be implemented.

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Achieving control at the quantum level is a pivotal requirement for the grounding of quantum technology and the development of reliable protocols for information processing. Frequently, state-manipulation of a quantum device needs the connection of remote nodes of a network and the creation of their entangled state. Such a *delocalized architecture* has received strong experimental attention, especially at the quantum optics level. Heralded entanglement of remote atomic ensembles or individually-trapped ions has been produced and atom-photon entanglement has been observed [1]. The transfer of prebuilt entanglement to distant systems has been proposed as a way to distribute quantum channels [2].

A different approach exploits a mediated interaction between two remote nodes, 1 and 2, by means of their sequential coupling to the same ancillary system  $e$ : The ancilla can bring to system 2 the information that has been previously impressed on it by its interaction with system 1. Recently, this idea has been used in a solid-state context involving multiple electron scattering between magnetic impurities [3, 4, 5]. Interestingly,  $e$  can also be used so as to condition the state of 1 and 2. Once a three-body correlated state is established by means of bilocal  $1 - e$  and  $2 - e$  interactions, by measuring the state of  $e$  we could project the remote systems onto entangled states with a non-zero probability [3, 4, 5, 6]. In these examples, 1 and 2 are embodied by two-level systems whose finite Hilbert space bounds the entanglement that can be shared [7]. Overcoming such a limitation is an important task deserving attention.

Here we present a scheme that allows the “extraction” of maximally entangled states via an effective non-demolition Bell measurement performed onto the state of two spin- $s$  particles. This occurs through repeated injection and post-selection of simple mediators, each undergoing multiple scattering and spin-flipping between the two spins [8]. Besides achieving the maximum number of ebits allowed to two spin- $s$  systems, the protocol provides a procedure for accumulating entanglement. Remarkably, our protocol does not require interaction-time tuning. In our scheme maximal entanglement is

stable against the parameters of the conditioned dynamics, which is a clear advantage in experimental implementations. In order to fix the ideas, we first describe the protocol in terms of a system composed of a conduction electron and two magnetic impurities. This will allow us to clearly illustrate the relevant features of our scheme. Later, we show how a cavity-quantum electrodynamics (QED) system, consisting of two multilevel atoms interacting with a photon field, can also embody the desired dynamics and allows a prompt experimental implementation. We consider a quasi one-dimensional (1D) wire, such as a semiconductor quantum wire [9] or a single-wall carbon nanotube [10], where two identical spin- $s$  magnetic impurities 1 and 2 are embedded at positions  $x_1 = 0$  and  $x_2 = x_0$  [see Fig. 1(a)]. Left-incident single electrons undergo multiple scattering between the two impurities and simultaneous spin-flipping. Assuming that the electron's coherence length exceeds  $x_0$  and that each electron occupies only the lowest subband, the Hamiltonian reads (we set  $\hbar = 1$ )  $\hat{H} = \hat{p}^2/(2m^*) + J \hat{\sigma} \cdot [\hat{\mathbf{S}}_1 \delta(x) + \hat{\mathbf{S}}_2 \delta(x - x_0)]$ . Here,  $\hat{p} = -i\nabla$ ,  $m^*$  and  $\hat{\sigma}$  are the electron momentum, effective mass and Pauli spin operator respectively.  $\hat{\mathbf{S}}_i$  is the spin- $s$  operator of the impurity  $i = 1, 2$  and  $J$  is the Heisenberg exchange coupling constant whose dimensions are frequency times length. Due to the elastic nature of the interactions, the energy spectrum reads  $E = k^2/2m^*$  ( $k$  is the electron wavevector). We label with  $\hat{\mathbf{S}} = \hat{\sigma} + \hat{\mathbf{S}}_1 + \hat{\mathbf{S}}_2$  the total spin of the system, while  $m_i$  and  $m_e = \pm 1/2$  are the quantum numbers associated with  $\hat{S}_{iz}$  and  $\hat{\sigma}_z$ ,

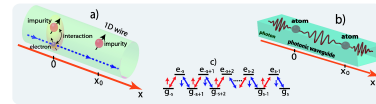


FIG. 1: (Color online) Setups for the implementation of our scheme in nanowire (a) and cavity-QED (b). (c) Multilevel atom embodying a spin- $s$  particle for the setup in panel (b), where symbols  $\uparrow, \downarrow$  indicate, abstractly, proper polarization of a photon.

respectively. From now on, we denote  $\{1/2, -1/2\}$  by  $\{\uparrow, \downarrow\}$  and, for convenience, we use the basis of product states  $|m_e, \{m_i\}\rangle = |m_e\rangle_e |m_1, m_2\rangle_{12}$ . We prepare the impurities in  $|\{m'_i\}\rangle_{12}$ . An incoming electron of wavevector  $k$  and spin state  $|m'_e\rangle_e$  is reflected (transmitted) in the state  $|m_e\rangle_e$ , while the impurities' spin state changes into  $|\{m_i\}\rangle_{12}$  with probability amplitude  $r(t)$  (we omit the dependence of  $r$  and  $t$  on  $m_{e(i)}$  and  $m'_{e(i)}$ ). As  $\hat{S}_z$  is a constant of motion, the only non-zero amplitudes are those obeying the selection rule  $m'_{12} + m'_e = m_{12} + m_e$  with  $m_{12} = m_1 + m_2$ . We solve this scattering problem by finding the steady states  $|k, m'_e, \{m'_i\}\rangle$  with input part  $\langle x | k, m'_e, \{m'_i\} \rangle_{in} = e^{ikx} \theta(-x) |m'_e, \{m'_i\}\rangle$ , where  $\theta(x)$  is the Heaviside step function. Their output part reads  $\langle x | k, m'_e, \{m'_i\} \rangle_{out} = \sum_{\alpha=r,t} \langle x | k, m'_e, \{m'_i\} \rangle_{\alpha}$  with  $\langle x | k, m'_e, \{m'_i\} \rangle_{\alpha} = \sum_{m_e, \{m_i\}} \alpha f_{\alpha}(x) |m_e, \{m_i\}\rangle$  and  $f_{\alpha}(x) = e^{i\eta_{\alpha} k x} \theta(\eta_{\alpha} x - \frac{1+\eta_{\alpha}}{2} x_0)$  ( $\eta_r = -\eta_t = -1$ ). The steady states are computed at all orders in  $J$  solving the time-independent Schrödinger equation and imposing the matching of the wavefunction at  $x_i$ 's [4]. We now derive how an (in general mixed) initial state of the impurities  $\rho_{12}$  is transformed after scattering of an electron incoming in an arbitrary statistical mixture  $\rho_e$  of the spin states  $|\uparrow\rangle_e$  and  $|\downarrow\rangle_e$ . To this aim, we consider the state having  $|k\rangle \langle k| \rho_e \rho_{12}$  as input part, where  $\langle x | k\rangle = e^{ikx} \theta(-x)$ . The output part of such state is found by expanding it in the basis  $\{|k, m'_e, \{m'_i\}\rangle$  and replacing each component of this expansion with the corresponding output part. A further projection onto the electron's position eigenstates far from the impurities  $|x_r\rangle$  and  $|x_t\rangle$  ( $x_r \ll 0$ ,  $x_t \gg x_0$ ) yields  $\sum_{\alpha=r,t} \langle x_{\alpha} | k, m'_e, \{m'_i\} \rangle_{\alpha} \langle k, m'_e, \{m'_i\} | x_{\alpha} \rangle |x_{\alpha}\rangle \langle x_{\alpha}|$ . After tracing over the electron's degrees of freedom, the impurities' state becomes

$$\mathcal{E}_{\rho_e}(\rho_{12}) = \sum_{\mu, \nu=\uparrow, \downarrow} \rho_{e\nu\nu} (\hat{R}_{\nu}^{\mu} \rho_{12} \hat{R}_{\nu}^{\mu\dagger} + \hat{T}_{\nu}^{\mu} \rho_{12} \hat{T}_{\nu}^{\mu\dagger}), \quad (1)$$

where  $\sum_{\mu} (\hat{R}_{\nu}^{\mu\dagger} \hat{R}_{\nu}^{\mu} + \hat{T}_{\nu}^{\mu\dagger} \hat{T}_{\nu}^{\mu}) = \mathbb{1}_{12}$ . Each Kraus operator  $\hat{R}_{\nu}^{\mu}$  ( $\hat{T}_{\nu}^{\mu}$ ) depends only on  $r$ 's ( $t$ 's) and is physically interpreted as the effect on  $\rho_{12}$  due to the detection in spin-state  $|\mu\rangle_e$  of a reflected (transmitted) electron incoming in state  $|\nu\rangle_e$ . We want to show that, conditioning the map in Eq. (1) and iterating it for  $n$  electrons (injected in succession in the same spin state), singlet-state extraction is efficiently performed. To achieve this, we first describe what is induced by post-selecting the state of  $n = 1$  scattered electrons. Preparation and post-selection of a given electron spin state, say  $|\uparrow\rangle_e$ , can be accomplished using spin-filtering contacts at the input/output ports of the wire [12], each selecting the same spin state. We obtain the final impurities' state  $\varrho_{12}^{(1)} = \mathcal{E}_{\uparrow\uparrow}^{(1)}(\rho_{12}) = (\hat{R}_{\uparrow}^{\uparrow} \rho_{12} \hat{R}_{\uparrow}^{\uparrow\dagger} + \hat{T}_{\uparrow}^{\uparrow} \rho_{12} \hat{T}_{\uparrow}^{\uparrow\dagger}) / P_{\uparrow\uparrow}^{(1)}(\rho_{12})$  with success probability  $P_{\uparrow\uparrow}^{(1)}(\rho_{12}) = \text{Tr}_{12}(\hat{R}_{\uparrow}^{\uparrow} \rho_{12} \hat{R}_{\uparrow}^{\uparrow\dagger} + \hat{T}_{\uparrow}^{\uparrow} \rho_{12} \hat{T}_{\uparrow}^{\uparrow\dagger})$ . The state  $\varrho_{12}^{(n)}$  corresponding to  $n$  electrons being prepared and post-

selected in  $|\uparrow\rangle_e$  is obtained as  $\varrho_{12}^{(n)} = \mathcal{E}_{\uparrow\uparrow}^{(n)}(\rho_{12})$  with conditional probability  $P_{\uparrow\uparrow}^{(n \geq 1)}(\rho_{12}) = \prod_{j=1, n} P_{\uparrow\uparrow}(\varrho_{12}^{(j-1)})$  and  $\varrho_{12}^{(0)} = \rho_{12}$  [11]. Here, the rate of electron-injection is chosen so that, as an electron reaches the impurities, the previous one has been already scattered off. Let  $|\Psi_s^{-}\rangle$  be the singlet state of two spin- $s$  impurities. Using resonance conditions (i.e.  $kx_0/\pi \in \mathbb{Z}$ ), in Fig. 2(a) and (b) we consider the case  $s=1/2$  and plot the fidelity  $F^{(n)}$  of  $\varrho_{12}^{(n)}$  with respect to the singlet  $|\Psi_{1/2}^{-}\rangle$  together with  $P_{\uparrow\uparrow}^{(n)}$  as functions of  $n$  and  $J/v$  for the initial product state  $|1/2, -1/2\rangle_{12}$  ( $v=k/m^*$  is the electronic group velocity). Clearly,  $F^{(n)} \rightarrow 1$  for a range of values around  $J/v \simeq 1.5$  that becomes a plateau when  $n$  increases ( $n < 7$  iterations are enough to get fidelity higher than 0.95). For a fixed value of  $J/v$ , such convergence is *exponential* in  $n$ . Remarkably, although our protocol is conditioned on the outcomes of  $n$  projective measurements all with the same outcome, the probability of success converges exponentially to 0.5. Differently from [3, 4, 5], the scheme is still efficient for a non-optimal  $J/v$ . Only a larger  $n$  is required, for a fixed  $s$ . Moreover, the process is robust against discrepancies of  $k$  with respect to resonance conditions and the use of a stream of mediators with mutually different wavevectors. In fact, by considering a Gaussian distribution of wave vectors centered at  $k$  with variance  $\sigma$ , we have found that the fidelity (probability) is larger than 0.9 (0.35) for  $kx_0 \in [0.9, 1.03]\pi$  and  $\sigma/k$  up to  $\simeq 5\%$ .

We now address the dependence of our figures of merit on the dimensionality of the impurities' spin. While the optimum ratio  $J/v$  depends slightly on  $s$ , the efficiency of singlet extraction persists, as shown in Fig. 2(c) for  $\rho_{12} = |s, -s\rangle \langle s, -s|$  with  $s = 1/2, 1, 3/2$ . Evidently,  $\varrho_{12}^{(n)}$  rapidly converges to the singlet state regardless of  $s$  (for instance,  $F^{(n>5)} > 0.95$  for  $s=1$ ) while  $P_{\uparrow\uparrow}^{(n)}$  approaches a *finite* value according to  $P_{\uparrow\uparrow}^{(n \gg 1)}(\rho_{12}) \rightarrow |\langle \Psi_s^{-} | s, -s \rangle|^2 = (2s+1)^{-1}$ , exponentially in  $n$ . Our scheme thus *asymptotically performs an effective projective measurement onto the spin- $s$  singlet state*. As the singlet state has the maximum number of ebits allowed by the dimension of the Hilbert space of each impurity, the scheme provides

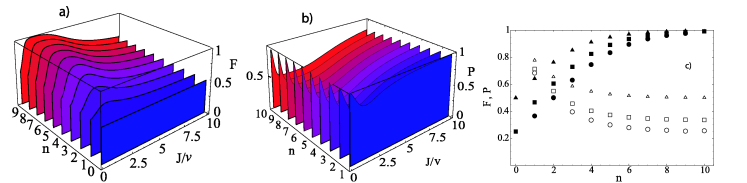


FIG. 2: (Color online) (a) and (b) Fidelity and success probability vs.  $J/v$  and  $n$  for  $s = 1/2$ . (c)  $F^{(n)}$  (filled symbols) and  $P_{\uparrow\uparrow}^{(n)}$  (empty symbols) vs.  $n$  for  $s = 1/2$  and  $J/v = 1.5$  ( $\blacktriangle, \triangle$ ),  $s = 1$  and  $J/v = 1.2$  ( $\blacksquare, \square$ ) and  $s = 3/2$  and  $J/v = 1.1$  ( $\bullet, \circ$ ) at  $kx_0/\pi \in \mathbb{Z}$  ( $J/v$  is optimized for each  $s$ ).

a way to extract more than one ebit by considering sufficiently high-dimensional impurities' spins. Moreover, an entanglement accumulation mechanism is achieved [7]. For instance, for  $s=2$  and  $J/v=1$  the impurities' entanglement (measured by the logarithmic negativity, which is upper-bounded by  $\log_2(d)$  for a  $d^2$ -dimensional Hilbert space) after  $n=2, 4$  and  $5$  is respectively,  $1.2, 1.8$  and  $2$ . These are larger than the bound given by  $\log_2(2s+1)$  for  $s=1/2, 1$  and  $3/2$ , making our system an iteratively exploitable quantum channel: The impurities' entanglement can be extracted to many pairs of qubits [7]. Similar results hold for any initial eigenstate of  $\hat{S}_{12z} = \hat{S}_{1z} + \hat{S}_{2z}$  with null eigenvalue.

We now show how the efficiency of singlet-state extraction relies on resonance-induced selection rules. Let  $|s, s, s_{12}, m_{12}\rangle$  be the coupled basis of common eigenstates of  $\hat{S}_1^2, \hat{S}_2^2, \hat{S}_{12}^2$  and  $\hat{S}_{12z}$  (the singlet state thus reads  $|\Psi_s^-\rangle = |s, s, s_{12}=0, m_{12}=0\rangle$ ). Let  $\mathcal{E}_\uparrow(\rho_{12})$  be the *unconditioned* map in Eq. (1) for  $\rho_e = |\uparrow\rangle_e \langle\uparrow|$ . Clearly, with the additional output-filtering of  $|\uparrow\rangle_e$ ,  $\mathcal{E}_\uparrow(\rho_{12})$  becomes  $\mathcal{E}_{\uparrow\uparrow}(\rho_{12})$ . Notice that in general the product state  $|s, s\rangle_{12}$  is the only fixed point of  $\mathcal{E}_\uparrow(\rho_{12})$ . However, at resonance ( $kx_0 = n\pi$ ),  $\hat{S}_{12}^2$  is conserved due to the equal probabilities of the electron to be found at each of  $x_i$ 's [4]. Thus, repeated applications of the unconditioned map cannot drive the system out of the eigenspace associated with a set value of  $s_{12}$ . This and the conservation of  $\hat{S}_z$  imply that the singlet state  $|\Psi_s^-\rangle$  becomes an additional fixed point of  $\mathcal{E}_\uparrow$ . Let  $p_{s_{12}}$  be the probability for an injected electron prepared in  $|\uparrow\rangle_e$  to be flipped down when the impurities are prepared in  $|s, s, s_{12}, 0\rangle$ . The selection rules at resonance yield the evolved impurities' state  $p_{s_{12}} |s, s, s_{12}, 1\rangle \langle s, s, s_{12}, 1| + (1-p_{s_{12}}) |s, s, s_{12}, 0\rangle \langle s, s, s_{12}, 0|$ . If we post-select  $|\uparrow\rangle_e$  at the output ports, each state  $|s, s, s_{12}, 0\rangle$  with  $s_{12} \neq 0$  is left unchanged with probability  $1 - p_{s_{12}}$ . Under application of  $\mathcal{E}_{\uparrow\uparrow}^{n \gg 1}$ , it thus vanishes as  $(1-p_{s_{12}})^{n \gg 1} \simeq 0$ , which clarifies the exponential convergence exhibited by  $F^{(n)}$  and  $P_{\uparrow\uparrow}^n$  (cf. Fig. 2). Differently,  $|s, s, s_{12}=0, 0\rangle = |\Psi_s^-\rangle$  survives to the application of  $\mathcal{E}_{\uparrow\uparrow}^{n \gg 1}$  since the selection rules ensure that  $p_{s_{12}=0} = 0$  [4]. If we consider an element of the uncoupled basis  $|\xi\rangle$  such that  $\hat{S}_{12z} |\xi\rangle_{12} = 0$  and expand it over  $|s, s, s_{12}, 0\rangle$ 's, we find that, under application of  $\mathcal{E}_{\uparrow\uparrow}^{n \gg 1}$ ,  $|\xi\rangle \langle \xi| \rightarrow |\Psi_s^-\rangle \langle \Psi_s^-|$  with a probability  $P_{\uparrow\uparrow}^{(n \gg 1)}$  that asymptotically becomes  $|\langle \Psi_s^- | \xi \rangle|^2$ . When  $|\xi\rangle = |s, -s\rangle_{12}$ , as in Fig. 2, the asymptotic probability is  $(2s+1)^{-1}$ . Our clear interpretation of the physics behind our protocol is an important feature for the development of novel schemes.

Unlike previous proposals [3, 4, 5], a remarkable advantage of our protocol is that it can be applied to magnetic impurities of spin higher than  $1/2$ . For instance, we could use a 1D semiconducting wire with embedded Mn impurities having  $s=5/2$ . Although impressive progresses have been made, a major obstacle in spintron-

ics implementations is the current lack of high-efficiency electron-spin filters [12]. As a way to overcome such difficulties, we discuss an alternative system [see Fig. 1(b)] able to act as an accurate simulator of  $\hat{H}$  and holding the promises for not far-fetched experimental implementation. The basic idea is to replace the electron with a single photon propagating in a 1D photonic waveguide sustaining two frequency-degenerate orthogonally polarized modes. For consistency of notation, we denote circular polarizations by  $\uparrow$  and  $\downarrow$ . Each impurity is now embodied by a multilevel atom [see Fig. 1(c)] having a  $(2s+1)$ -fold degenerate ground level spanned by  $\{|g_{-s}\rangle, \dots, |g_s\rangle\}$  and a  $2s$ -fold degenerate excited level spanned by  $\{|e_{-s}\rangle, \dots, |e_{s-1}\rangle\}$ . The standard three-level  $\Lambda$  and five-level  $M$  configurations are recovered, for instance, by taking  $s=1/2$  and  $s=1$ , respectively. Such a configuration may be found in the rich hyperfine spectrum of alkali atoms. We assume electric-dipole selection rules such that each  $|e_m\rangle$  ( $m=-s, \dots, s-1$ ) is connected to the pair of nearest-neighbor ground states  $\{|g_m\rangle, |g_{m+1}\rangle\}$  via coherent scattering of a photon between the two orthogonally polarized modes. To fix the ideas, we take the transition  $|e_m\rangle \leftrightarrow |g_m\rangle$  ( $|e_m\rangle \leftrightarrow |g_{m+1}\rangle$ ) to be driven by the  $\uparrow$ -polarized ( $\downarrow$ -polarized) mode. Each atom can thus undergo a transition between two adjacent ground states  $|g_m\rangle \leftrightarrow |g_{m+1}\rangle$  via a two-photon Raman process with associated coherent scattering of a photon between states  $|\uparrow\rangle$  and  $|\downarrow\rangle$ . Assuming a linear dispersion law  $E = v_{ph}k$  with  $v_{ph}$  the group velocity of the photon and  $E$  its energy, the free Hamiltonian of the field in the waveguide is [13]  $\hat{H}_{ph} = -i \sum_{\beta=R,L} \sum_{\gamma=\uparrow,\downarrow} \int dx v_\beta \hat{c}_{\beta,\gamma}^\dagger(x) \partial_x \hat{c}_{\beta,\gamma}(x)$  with  $v_R = -v_L = v_{ph}$  and  $\hat{c}_{R,\gamma}^\dagger(x)$  [ $\hat{c}_{L,\gamma}^\dagger(x)$ ] the bosonic operator creating a right (left) propagating photon of polarization  $\gamma$  at position  $x$ . Considering dipole transitions with Rabi frequencies and natural excited-state linewidth smaller than the corresponding detuning from the excited state, each state  $|e_m\rangle$  is only virtually populated and the effective atom-photon coupling reads  $\hat{V} = \sum_{i=1,2} \int dx \hat{c}_\uparrow^\dagger(x) \hat{c}_\downarrow(x) \hat{S}_{i-} + \text{h.c.} \delta(x - x_i)$  with  $\hat{c}_\uparrow^\dagger(x) = \sum_{\beta=R,L} \hat{c}_{\beta,\uparrow}^\dagger(x)$  and  $\hat{S}_{i+} = \hat{S}_{i-}^\dagger = \sum_{m=-s}^{s-1} J_{s,m} |g_{m+1}\rangle_i \langle g_m|$ . Here  $J_{s,m}$  is the effective transition rate of the Raman process leading the  $i$ -th atom from  $|g_m\rangle_i$  to  $|g_{m+1}\rangle_i$ , assuming identical atoms. We map the photonic polarization into an effective pseudospin- $s$  as  $\hat{\sigma} = \int dx \hat{\sigma}(x)$  with  $\hat{\sigma}_+(x) = \hat{\sigma}_-^\dagger(x) = \hat{c}_\uparrow^\dagger(x) \hat{c}_\downarrow(x)$  and  $\hat{\sigma}_z(x) = [\hat{c}_\uparrow^\dagger(x) \hat{c}_\uparrow(x) - \hat{c}_\downarrow^\dagger(x) \hat{c}_\downarrow(x)]/2$ . Provided that  $J_{s,m} = J \chi_{s,m}$  with  $\chi_{s,m} = [s(s+1) - m(m+1)]^{1/2}$ , each  $\hat{S}_{i\pm}$  becomes the effective pseudospin- $s$  operator  $\hat{S}_{i\pm} = J \hat{S}_{i\pm}$ , where  $\hat{S}_{i\pm}$  obeys the standard algebra of angular momentum. Under these conditions, this model can be regarded as the second quantization version of  $\hat{H}$  with the exchange electron-impurity coupling replaced by an isotropic XY interaction. It is easily checked that  $[\hat{H}_{ph} + \hat{V}, \hat{S}_z] = 0$  and, provided  $kx_0/\pi \in \mathbb{Z}$ ,  $[\hat{H}_{ph} + \hat{V}, \hat{S}_{12}^2] = 0$ . Through

standard procedures [13], we have derived the stationary states  $|k, m'_{ph}, \{m'_i\}\rangle$  for a single photon with wavevector  $k$  ( $m'_{ph}$  is the quantum number of  $\hat{\sigma}_z$ ). The input (output) part of  $|k, m'_{ph}, \{m'_i\}\rangle$  is formally analogous to  $|k, m'_e, \{m'_i\}\rangle_{in}$  ( $|k, m'_e, \{m'_i\}\rangle_{out}$ ). Here,  $\mathcal{E}_{\uparrow\uparrow}(\rho_{12})$  is obtained analogously to what is done for the previous model with photonic polarization detection used for the post-selection. Plots analogous to those in Figs. 2 are reproduced with only negligible quantitative differences. Practically,  $\mathcal{E}_{\uparrow}(\rho_{12})$  is obtained using Geiger-like photodetectors at the input/output ports of the waveguide combined with polarizing beam-splitters to realize  $\mathcal{E}_{\uparrow\uparrow}(\rho_{12})$ . Each  $J_{s,m}$  depends on the product of the Clebsch-Gordan coefficients associated with the far-detuned (one-photon) transitions involved in the process  $|g_m\rangle \leftrightarrow |g_{m+1}\rangle$ . The condition  $J_{s,m} = J \chi_{s,m}$  is clearly fulfilled for  $s=1/2$ , involving only  $\chi_{1/2,-1/2}=1$ . For  $s \geq 1$  the pattern of  $J_{s,m}$ 's might in general deviate from the ideal one dictated by the  $\chi_{s,m}$ 's. However, we have assessed  $F^{(n)}$  and  $P_{\uparrow\uparrow}^n$  finding that our scheme is strikingly robust against such deviations [14]. For instance, for  $s=3/2$ , the ideal pattern yields  $J_{3/2,1/2}/J_{3/2,-3/2} = 1$  and  $J_{3/2,-1/2}/J_{3/2,-3/2} = 2/\sqrt{3}$ . By taking  $J_{3/2,-3/2}/v_{ph} = J_{3/2,1/2}/v_{ph} = \sqrt{3}$  and  $J_{3/2,-1/2}/v_{ph} = 4\sqrt{3}$ , which are far from ideal, we obtain  $F^{(n>6)} = 0.97$ , and  $P^{(n>6)} = 0.26$ . These values are basically identical to the values obtained with the ideal ratios. This alternative model turns out to be also robust against deviations of  $k$  from the ideal resonance conditions [14]. Our protocol is thus resilient and flexible to the actual working conditions.

For a realization of the scheme in the case  $s=1/2$ , the impurities can be embodied by  $\Lambda$  configurations encompassed in the (single-electron charged) trionic picture of semiconducting quantum dots (QDs), which have been the center of extensive studies [15]. Positioning QDs within a waveguide or a cavity is now achievable with high accuracy ( $\sim 30\text{nm}$ ). It can be easily shown that for a photonic wavelength of 780nm in a GaAs structure (400nm in a GaN nanowire),  $x_0 \sim 0.1\mu\text{m}$  ( $1\mu\text{m}$ ) is required for the resonance condition, which is achievable. Strong coupling between a single QD and a cavity field has been demonstrated [15] and current experimental efforts make the achievement of  $J/v \sim 1$  realistic in large refractive-index structures, without the need of waveguide's bandgap. We consider GaInN (InAs) QDs in GaN (GaAs) nanowires as potential candidates for our scheme. Their typical quality factor is  $\simeq 10^3$ , implying single-photon lifetime  $\tau_p \sim 1\text{ps}$  at 400nm wavelength. The refractive index of GaN is  $\sim 2$ , so that a photon travels  $x_0 = 1\mu\text{m}$  in  $\tau_p/100$ . Ongoing experimental progresses make the controlled growth and positioning of two QDs in  $\mu\text{m}$ -long waveguides, quite realistic.

We have proposed a scheme for the conditional ex-

traction of singlet states of two remote spin- $s$ 's based on projective measurements over interaction mediators. The protocol does not require the demanding recycling of the same mediator. It achieves  $s+1/2$  ebits with finite probability, a small number of steps, weak requirements on the parameters entering the dynamics and no interaction-time tuning. We have proposed a realistic setup where the mediators are embodied by photons and the spins to be entangled by artificial atoms.

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